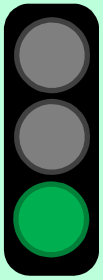
Look at the following equation:

$$a + b = c$$

How many positive **integer** solutions can you find for a, b and c?

whole number



Challenge

How many solutions are there in total? Explain your answer.

$$a + b = c$$

Here are a few examples:

$1 + 2 = 3$ $\quad a = 1, b = 2, c = 3$

$4 + 3 = 7$ $\quad a = 4, b = 3, c = 7$

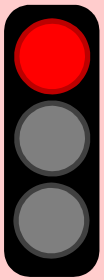
$1 + 1 = 2$ $\quad a = 1, b = 1, c = 2$

Now look at this equation. What has changed?

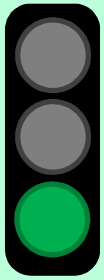
$$a^2 + b^2 = c^2$$

How many positive **integer** solutions can you find for a, b and c?

whole number



Stuck?
Try writing out a list of square numbers first.
You can use a calculator for squares bigger than 15.



Challenge
Can you group your sets of solutions in any way?
Is there an easy way of generating multiple solutions quickly?

$$a^2 + b^2 = c^2$$

Here are two solutions for this equation:

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

$a = 3$
$b = 4$
$c = 5$

$$6^2 + 8^2 = 36 + 48 = 100 = 10^2$$

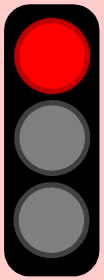
$a = 6$
$b = 8$
$c = 10$

What is the link between these two solutions?

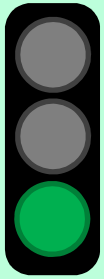
Now look at this equation. What has changed?

$$a^3 + b^3 = c^3$$

How many positive **integer** solutions can you find for a, b and c?



Stuck?
Try writing out a list of cube numbers first. You can use a calculator for cubes bigger than 5.



Challenge
Got an answer?
Write an explanation to convince a partner that you are correct.

Pierre de Fermat
(1601 – 1665)
French

Arithmeticorum Liber II. 61

interuallum numerorum 2. minor autem 1 N. atque ideo maior 1 N. + 2. Oportet itaque 4 N. + 4. triplos esse ad 3. & adhuc superaddere 10. Ter igitur 2. adscitis vnitatibus 10. æquatur 4 N. + 4. & fit 1 N. 3. Erit ergo minor 3. maior 5. & satisfaciunt quæstioni.

IN QVÆSTIONEM VII.

CONDITIONIS appositæ eadem ratio est quæ & appositæ præcedenti quæstioni, nil enim aliud requirit quàm vt quadratus interualli numerorum sit minor interuallo quadratorum, & Canones iidem hic etiam locum habebunt, vt manifestum est.

QVÆSTIO VIII.

PROPOSITVM quadratum diuidere in duos quadratos. Imperatum sit vt 16. diuidatur in duos quadratos. Ponatur primus 1 Q. Oportet igitur 16 – 1 Q. æquales esse quadrato. Fingo quadratum à numeris quotquot libuerit, cum defectu tot vnitatum quod continet latus ipsius 16. esto 2 2 N. – 4. ipse igitur quadratus erit 4 Q. + 16. – 16 N. hæc æquabuntur vnitatibus 16 – 1 Q. Communis adiciatur vtrimque defectus, & à similibus auferantur similia, fient 5 Q. æquales 16 N. & fit 1 N. $\frac{16}{5}$. Erit igitur alter quadratorum $\frac{256}{25}$. alter verò $\frac{144}{25}$ & vtriusque summa est $\frac{400}{25}$ seu 16. & vterque quadratus est.

OBSERVATIO DOMINI PETRI DE FERMAT.

CVbum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum vltrà quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

QVÆSTIO IX.

RVRSVS oporteat quadratum 16 diuidere in duos quadratos. Ponatur rursus primi latus 1 N. alterius verò quotcunque numerorum cum defectu tot vnitatum, quot constat latus diuidendi. Esto itaque 2 N. – 4. erunt quadrati, hic quidem 1 Q. ille verò 4 Q. + 16. – 16 N. Cæterum volo vtrumque simul æquari vnitatibus 16. Igitur 5 Q. + 16. – 16 N. æquatur vnitatibus 16. & fit 1 N. $\frac{8}{5}$ erit

H iii

"… to … into … a … to … rs, … ny … an … two … ve … ly … of … this, which this margin is too narrow to contain…"

Pierre de Fermat
(1601 – 1665)
French

"It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain..."